Technical Report: Optimal Resource Allocation in Cognitive Smart Grid Networks

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In this technical report, we explain the fault tolerance property thresholds in *Spread Spectrum Communication (SSC)* that we assumed in our proposed *Cognitive Smart Grid Network (CSGN)*. Garg [1] shows that DS-CDMA receivers will interpret any amplitude between [0+ε,$\mathcal{A}$] as 0 and [-$\mathcal{A}$,$0-\varepsilon$] as 1. Hence, their scheme will tolerate up to length of OCSs minus noise power, or $L-\varepsilon$, fault without considering idle senders (As we will show, L is proportional to $\mathcal{A}$). Gerakoulis [2] also elaborates that orthogonal Gold codes can tolerate up to 50% of the OCS length of jitter. We are assuming that the sender can be silent at times. Thus, our scheme consists of three voltage levels representing the sender sending 1, nothing, and -1 respectively. Based on what we will show in this analysis, the receiving node will be able to tolerate an error of lower than 37.5% (of the received decoded value) in asynchronous and 50% in synchronous transmissions.

A. Fault tolerance threshold in synchronous CSGNs

In the following equations in this section, $C(t)$ is the channel state in the synchronous CSGN at time t and b_i represents the desperad signal using i^{th} OCS by a smart appliance in the synchronous CSGN. Also, $\eta(t)$ represents AWGN at time instant t. The $\mathcal{A}_i(t)$ and $\mathcal{A}_g(t)$ represent the amplitude share of the data spread by the i^{th} OCS and the g^{th} sub-OCS, respectively at time t. Let b_i represent the received despread decoded bit in the i^{th} receiver. $OCS_i(t)$ shows the t^{th} spreading chip in the i^{th} OCS.

$$C(t) = \sum_{i=1}^{L} \mathcal{A}_i(t) + \sum_{g=1}^{NSU=(log_4 L)-1} \mathcal{A}_g(t) + \eta(t) \quad (1)$$

$$b_i = \sum_{t=1}^{OCSLength=L} C(t).OCS_i(t). \quad (2)$$

Let $Q^*(t)$ represent the defined threshold of the required $\mathcal{A}$ in a synchronous CSGN at time t. Thus, in Eqn. 3 $\mathcal{A}$ represents the maximum signal amplitude (voltage) in the receiver that will be proportional to L or length of the OCSs in the maximum state when all appliances in CSGN are using their OCSs. Maximum peak-to-peak amplitude can be $2\mathcal{A}$ while $\mathcal{A}$ is proportional to L. Thus, if we want to define a threshold, we can decode the despread bit b_i to 1 in the i^{th} receiver, if b_i is greater than $\frac{\mathcal{A}}{2}$, and to -1, if b_i is smaller than $\frac{-\mathcal{A}}{2}$. $Q^*(t)$ represents this threshold in a synchronous transmission when the receiver is able to decide for choosing 1 or -1 where the despread waveform is greater or smaller than 50% of the maximum amplitude $\mathcal{A}$. Figure 1-a illustrates the case of the synchronous CSGN.

$$if \begin{cases} \frac{+\mathcal{A}}{2} < b_i < +\mathcal{A} & b_i = +1 \\ \frac{-\mathcal{A}}{2} \leq b_i \leq \frac{+\mathcal{A}}{2} & b_i = 0 \\ -\mathcal{A} < b_i < \frac{-\mathcal{A}}{2} & b_i = -1 \end{cases} \quad (3)$$

Hence, based on Eqn. 3, in order to decode a despread signal correctly in synchronous CSGN, the maximum fault that can be tolerated at the receiver in terms of $\mathcal{A}$ is:

$$Q^*(t) = |1 - b_i| \leq \frac{\mathcal{A}}{2}. \quad (4)$$

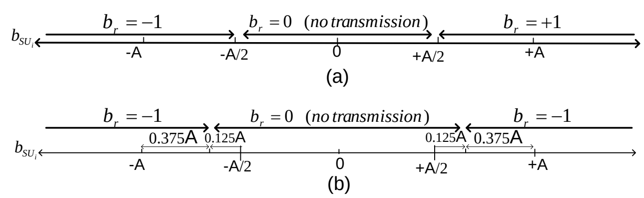

Fig. 1. Convert received despread amplitude to bit in receiver in a) synchronous CSGN, b) asynchronous CSGN.

B. Fault tolerance threshold in asynchronous CSGNs

However, in an asynchronous mode shown in Fig. 1-b, using the proposed sub-OCSs will cause, at most, $\frac{\mathcal{A}}{4}$ of the amplitude to be altered in a time instant. This will affect both positive and negative amplitudes equally ($\frac{\mathcal{A}}{8}$ or $0.125\mathcal{A}$ in positive voltages and $\frac{\mathcal{A}}{8}$ or $0.125\mathcal{A}$ in negative voltages). This effect, which is indicated by the interferer voltage by asynchronous OCS and sub-OCSs at time instant t in the figure, makes the acceptable decoding amplitude interval smaller, from 50% in the synchronous mode, to 37.5% in the asynchronous mode.

In the asynchronous CSGN, all the symbols and notations used are similar to the synchronous case except for the term $\mathcal{A}_p(t)$ that represents the interference caused by asynchronous transmission, where N_{asy} represents the number of nodes causing jitter interference. Throughout the paper, the noise power is assumed to be 1 (Fig. 1-b).

$$C(t) = \sum_{i=1}^{L} \mathcal{A}_i(t) + \sum_{g=1}^{NSU=(log_4 L)-1} \mathcal{A}_g(t) + \sum_{p=1}^{N_{asy}} \mathcal{A}_p(t) + \eta_t \quad (5)$$

$$b_i = \sum_{t=1}^{OCSLength=L} C(t).OCS_i(t). \quad (6)$$

Let $f_{b_i}^{min}$ denote the minimum required amplitude in an asynchronous receiver to decode the data bits from the despread signal correctly. Then,

$$f_{b_r}^{min} = b_i + \sum_{p=1}^{N_{asy}} \mathcal{A}_p(t) = 0.5\mathcal{A} + 0.125\mathcal{A} = 0.50\mathcal{A} + 0.125\mathcal{A} = 0.625\mathcal{A}. \quad (7)$$

Let $Q^*(t)$ denote the maximum fault tolerance in an synchronous transmission. Then,

$$if \begin{cases} 0.625\mathcal{A} < b_i < +\mathcal{A} & b_i = +1 \\ -0.625\mathcal{A} \leq b_i \leq +0.625\mathcal{A} & b_i = 0 \\ -\mathcal{A} < b_i < -0.625\mathcal{A} & b_i = -1 \end{cases} \quad (8)$$

Thus, based on Eqn. 8, considering time shift and jitter in OCSs and sub-OCSs, a despread signal can be decoded at a receiver in an asynchronous CSGN, if the fault is bounded by Eqn. 9 in terms of $\mathcal{A}$.

$$Q^*(t) = |1 - b_i| \leq 0.375\mathcal{A}. \quad (9)$$

To bring up a tangible example, assume that sending a data symbol with an OCS of length L results in an amplitude of $\mathcal{A}$ or $-\mathcal{A}$ at the receiver. Then, any corruption in a pattern of L will result in a change in $\mathcal{A}$, relative to the corrupted fraction of L. There is a linear relation between L and $\mathcal{A}$. Additionally, as we have seen, there is a way for the primary and secondary receivers to decode the received signal if the total noise and interference result in atmost 37.5% corruption of the decoded waveform (or 37.5% of the chip length), based on the fault-tolerance threshold presented above. In other words, if the decoded value (the value which is decoded at the receiver from the received signal) is between ($\mathcal{A} - 0.375\mathcal{A}$, $\mathcal{A} + 0.375\mathcal{A}$), it will be interpreted as $\mathcal{A}$ providing an error tolerance of 37.5% of the OCS length. For instance, if $\mathcal{A}$ = 256, any decoded value at the receiver ranging from ($\mathcal{A} - 0.375\mathcal{A}$)= 160 to ($\mathcal{A} + 0.375\mathcal{A}$) = 352 will be interpreted as 256 at the receiver. Likewise, any decoded value ranging from $(-\mathcal{A} - 0.375\mathcal{A})$ = -352 to $(-\mathcal{A} + 0.375\mathcal{A})$ = -160 will be interpreted as -256 at the receiver. In case of getting a decoded value between -160 to 160, the receiver will ignore the received data (equivalent to receiving 0) assuming that the sender has been silent (the received signal could be generated due to environmental noise). It should be noted that any value more than ($\mathcal{A} + 0.375\mathcal{A}$) or less than (-$\mathcal{A} - 0.375\mathcal{A}$) will be mapped to $+\mathcal{A}$ and $-\mathcal{A}$, respectively. It is worth mentioning that similar orthogonal codes, such as Gold, can also tolerate a timing jitter of 50% of the OCS length [2].

References

[1] V. k. Garg, *Wireless Communication and Networking*. Morgan Kaufmann Publishers, 2007.

[2] D. Gerakoulis and E. Geraniotis, *CDMA Access and Switching for Terrestrial and Sattelite Networks*. John Wiley and Sons, 2001.